

Closing Fri: 3.5(1)(2)

Closing *Tues*: 3.6-9

Closing next *Thur*: 3.9

Entry Task: Consider $y^3 + x^2 = 4$.

1. Find $\frac{dy}{dx}$

2. Find $\frac{d^2y}{dx^2}$

3.6 Logarithmic Derivatives

Recall logarithm facts:

$$1. y = \ln(x) \quad \leftrightarrow \quad e^y = x$$

$$y = \log_a(x) \quad \leftrightarrow \quad a^y = x$$

$$2. e^{\ln(x)} = x \quad \text{and} \quad \ln(e^y) = y$$

$$a^{\log_a(x)} = x \quad \text{and} \quad \log_a(a^y) = y$$

$$3. \ln(ab) = \ln(a) + \ln(b)$$

$$\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$$

$$\ln(x^n) = n \ln(x)$$

Test of basic understanding

a) Solve $3^x + 1 = 11$

b) Solve $(\log_5(2x) - 4)^3 = 8.$

Find the derivative of $y = \ln(x)$

Find the derivative of $y = \log_a(x)$

Example: Find the derivative of

a) $y = \ln(x^2 - 3x)$

b) $y = \tan^{-1}(2x) \ln(3x + 1)$

Power functions:

$$\frac{d}{dx} [(g(x))^n] = n(g(x))^{n-1} g'(x)$$

Example:

$$\frac{d}{dx} [(x^3 + 2x)^{10}] =$$

Exponential functions:

$$\frac{d}{dx} [e^{g(x)}] = e^{g(x)} g'(x)$$

$$\frac{d}{dx} [a^{g(x)}] = a^{g(x)} \ln(a) g'(x)$$

Examples:

$$\frac{d}{dx} [e^{(x^4 - 5x)}] =$$

$$\frac{d}{dx} [7^{(x^4 - 5x)}] =$$

What if x is in base AND exponent?

Example: $y = (3x + 1)^x$

Answer: Logarithmic Differentiation

Step 1: Take log of both sides

Step 2: Differentiate implicitly

Step 3: Solve for y' .